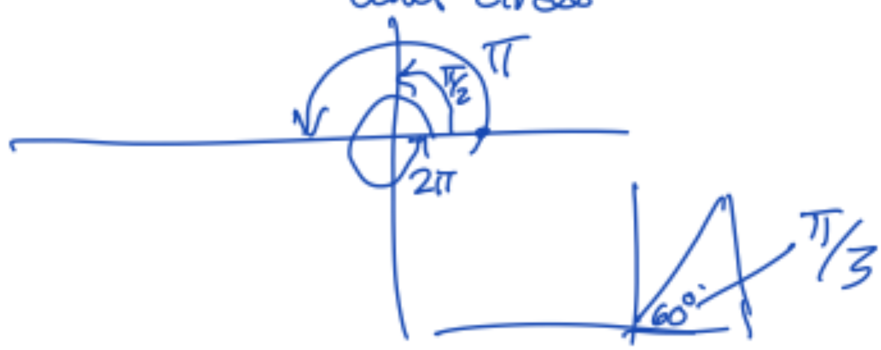
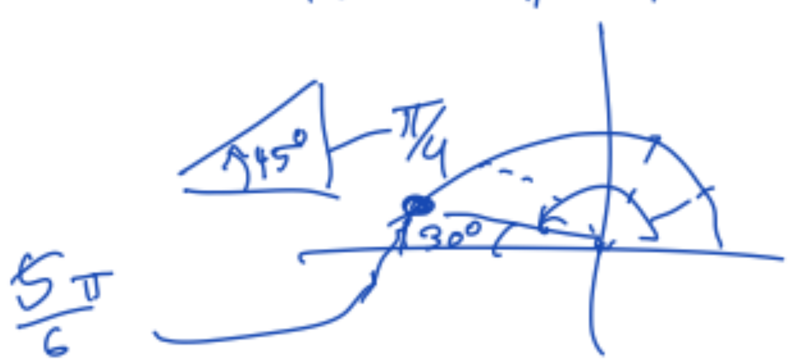
TRIG Review

unit circle(s) & angles — always measured from + x axis

$x^2+y^2=1$

θ — in radians — arclength along the unit circle.



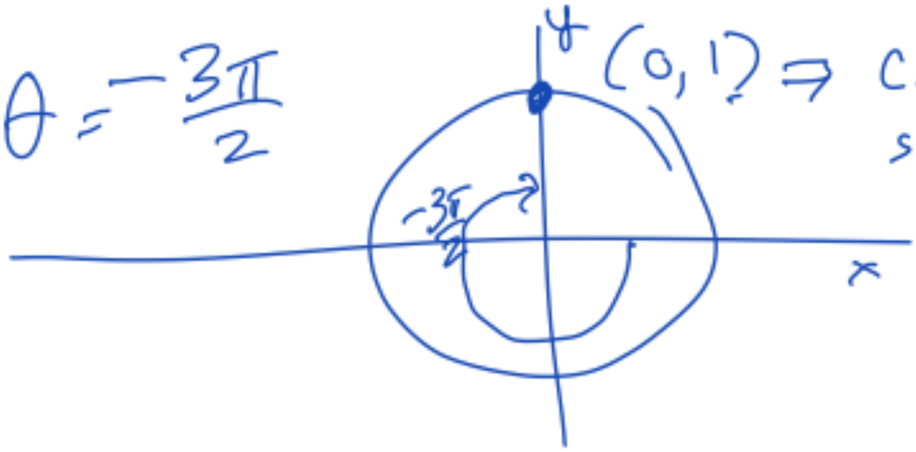


Angle $\theta \longrightarrow$ point (x,y) on S^1

<u>Trig Fcns:</u> $x = \cos(\theta)$

$y = \sin(\theta)$

$\theta = -\frac{3\pi}{2}$



$(0,1) \Rightarrow \cos\left(-\frac{3\pi}{2}\right) = 0$

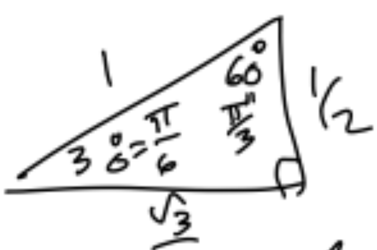
$\sin\left(-\frac{3\pi}{2}\right) = 1$

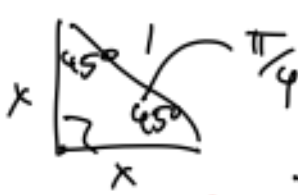
<u>Special Triangles:</u>

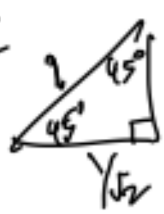
equilateral triangle

$h = \sqrt{1 - \left(\frac{1}{2}\right)^2}$

$h = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$







$1/\sqrt{2} = \frac{\sqrt{2}}{\sqrt{2}} \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$x^2 + x^2 = 1$

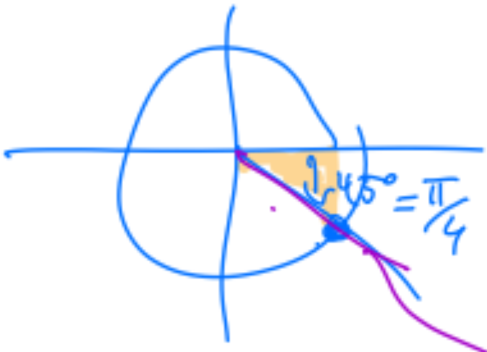
$x^2 = 1/2 \Rightarrow x = \frac{1}{\sqrt{2}}$

$$\cos\left(\frac{7\pi}{6}\right) = \frac{-\sqrt{3}}{2}$$

$$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$$

$$\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$



All trig fns

$x = \cos\theta$

$y = \sin\theta$

$\frac{y}{x} = \tan\theta$ = slope of the line to (x,y) from origin

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{x}$

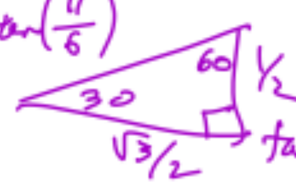
$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{y}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$.

$$\tan\left(\frac{-\pi}{4}\right) = -1$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

$\tan\left(\frac{\pi}{6}\right)$



$\tan\left(\frac{\pi}{6}\right) = \frac{1/2}{\sqrt{3}/2}$

$(-1,0)$

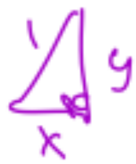
$\tan(\pi) = \boxed{0}$

$\frac{0}{-1} = 0$

$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

Trig identities

$x^2 + y^2 = 1$ unit circle



$$\boxed{\cos^2(\theta) + \sin^2(\theta) = 1}$$

Pythagorean.

divide by $\cos^2(\theta)$

$$\frac{\cos^2(\theta)}{\cos^2(\theta)} + \frac{\sin^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$$

$$\boxed{1 + \tan^2(\theta) = \sec^2(\theta)}$$

divide by $\sin^2\theta$

$$\frac{\cos^2}{\sin^2} + \frac{\sin^2}{\sin^2} = \frac{1}{\sin^2}$$

$$\boxed{\cot^2(\theta) + 1 = \csc^2(\theta)}$$

Toughones:

Angle Sum

$$\boxed{\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)}$$

$$\boxed{\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)}$$

$$\sin\left(\theta+\frac{\pi}{6}\right) = \sin(\theta)\cos\left(\frac{\pi}{6}\right) + \cos(\theta)\sin\left(\frac{\pi}{6}\right)$$

$$\boxed{\sin\left(\theta+\frac{\pi}{6}\right) = \sin(\theta)\left(\frac{\sqrt{3}}{2}\right) + \cos(\theta)\frac{1}{2}}$$

Double Angle: $\sin(A+A) = \sin(A)\cos(A) + \cos(A)\sin(A)$

$$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$$

$$\cos(A+A) = \cos(A)\cos(A) - \sin(A)\sin(A)$$

(1) $$\boxed{\cos(2A) = \cos^2(A) - \sin^2(A)}$$

$$\cos^2(A) = 1 - \sin^2(A)$$

$$\sin^2(A) = 1 - \cos^2(A)$$

(2) $$\boxed{\cos(2A) = 1 - 2\sin^2(A)}$$

(3) $$\boxed{\cos(2A) = 2\cos^2(A) - 1}$$

$$\boxed{\begin{aligned} \sin^2(A) &= \tfrac{1}{2}\left(1-\cos(2A)\right) \\ \cos^2(A) &= \tfrac{1}{2}\left(1+\cos(2A)\right) \end{aligned}}$$

$\arcsin\left(\frac{1}{2}\right)$ = angle whose $\sin(\ )$ is $\frac{1}{2}$

$= \frac{\pi}{6}$.

$$\cos(x) - \cos^2(x) - \sin^2(x)$$
$$= \cos(x) - \underbrace{\left[\cos^2(x) + \sin^2(x)\right]}_{1}$$

$$\frac{1}{\cos(x)-1} = \frac{1\,(\cos(x)+1)}{(\cos(x)-1)(\cos(x)+1)}$$

$$= \frac{\cos(x)+1}{\cos^2(x)-1}$$

$$-\sin^2(x)$$

$\sin^2(x)+\cos^2(x)=1$